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**Exercises**

**Section 4.1: 4, 10, 17, 20, 21, 23, 34, 35**

4. Verify that the polynomials p(x) = 5x 3 − 27x 2 + 45x − 21, q(x) = x 4 − 5x 3 + 8x 2 − 5x + 3

interpolate the data 

and explain why this does not violate the uniqueness part of the theorem on existence of polynomial interpolation.

**Answer:**

p(x) = 5x 3 − 27x 2 + 45x − 21,

p(1) = 2

p(2) = 1

p(3) = 6

p(4) = 47

q(x) = x 4 − 5x 3 + 8x 2 − 5x + 3

q(1) = 2

q(2) = 1

q(3) = 6

q(4) = 47

Hence: It does not violate the uniqueness, part of the existence theorem because two polynomials are not of same degree.

10. a. Construct Newton’s interpolation polynomial for the data shown.



b. Without simplifying it, write the polynomial obtained in nested form for easy evaluation.

Answer:

a.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| x | f[ ] | f[ , ] | f[ , , ] | f[ , , , ] |
| 0 | 7 | f[0,2] = (f[2] – f[0])/ (2 -0) = 2 | f[0,2,3] = (f[2,3]- f[0,2])/ (3-0) = 5 | f[0,2,3,4] = (f[2,3,4] – f[0,2,3]) / (4-0) = 1 |
| 2 | 11 | f[2,3] = (f[3] – f[2])/ (3 -2) = 17 | f[2,3,4] = (f[3,4]- f[2,3])/ (4-2) = 9 |  |
| 3 | 28 | f[3,4] = (f[4] – f[3])/ (4 -3) = 35 |  |  |
| 4 | 63 |  |  |  |

P3(x) = 7 + 2(x - 0) + 5(x -0)(x-2) + 1(x-0)(x-2)(x-3)

= x3 -2x + 7

b. P3(x) = 7 + 2(x - 0) + 5(x -0)(x-2) + 1(x-0)(x-2)(x-3)

= 7 + (x - 0) (2 + 5(x-2) + (x-2)(x-3))

= 7 + (x - 0) (2 + (x - 2)(5 + (x-3)))

= 7 + x(2 + (x-2)(5 + (x -3)))

17. Determine by two methods the polynomial of degree 2 or less whose graph passes through the points (0, 1.1), (1, 2), and (2, 4.2). Verify that they are the same

Answer:

|  |  |  |  |
| --- | --- | --- | --- |
| x | f[ ] | f[ , ] | f[ , , ] |
| 0 | 1.1 | f[0,1] = (f[1] – f[0])/ (1 -0) = 0.9 | f[0,1,2] = (f[1,2] – f[0,1])/ (2 -0) = 0.65 |
| 1 | 2 | f[1,2] = (f[2] – f[1])/ (2 -1) = 2.2 |  |
| 2 | 4.2 |  |  |

P(x) = 1.1 + 0.9 (x - 0) + 0.65 (x – 0) (x -1)

= 1.1 + 0.25x + 0.65x2

20. Without using a divided-difference table, derive and simplify the polynomial of least degree that assumes these values:

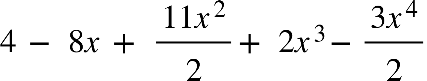


Answer:

l subscript 0 open parentheses x close parentheses equals fraction numerator open parentheses x space minus space x subscript 1 close parentheses. open parentheses x space minus space x subscript 2 close parentheses. space open parentheses x space minus space x subscript 3 close parentheses. open parentheses x space minus space x subscript 4 close parentheses over denominator open parentheses x subscript 0 minus x subscript 1 close parentheses. space open parentheses x subscript 0 minus x subscript 2 close parentheses. open parentheses x subscript 0 minus x subscript 3 close parentheses. open parentheses x subscript 0 minus x subscript 4 close parentheses end fraction space equals space fraction numerator open parentheses x space plus 1 close parentheses open parentheses x close parentheses open parentheses x minus 1 close parentheses open parentheses x minus 2 close parentheses over denominator 24 end fraction
l subscript 1 open parentheses x close parentheses equals fraction numerator open parentheses x space minus space x subscript 0 close parentheses. open parentheses x space minus space x subscript 2 close parentheses. space open parentheses x space minus space x subscript 3 close parentheses. open parentheses x space minus space x subscript 4 close parentheses over denominator open parentheses x subscript 1 minus x subscript 0 close parentheses. space open parentheses x subscript 1 minus x subscript 2 close parentheses. open parentheses x subscript 1 minus x subscript 3 close parentheses. open parentheses x subscript 1 minus x subscript 4 close parentheses end fraction space equals space fraction numerator negative open parentheses x space plus 2 close parentheses open parentheses x close parentheses open parentheses x minus 1 close parentheses open parentheses x space minus 2 close parentheses over denominator 6 end fraction
l subscript 2 open parentheses x close parentheses equals fraction numerator open parentheses x space minus space x subscript 0 close parentheses. open parentheses x space minus space x subscript 1 close parentheses. space open parentheses x space minus space x subscript 3 close parentheses. open parentheses x space minus space x subscript 4 close parentheses over denominator open parentheses x subscript 2 minus x subscript 0 close parentheses. space open parentheses x subscript 2 minus x subscript 1 close parentheses. open parentheses x subscript 2 minus x subscript 3 close parentheses. open parentheses x subscript 2 minus x subscript 4 close parentheses end fraction space equals space fraction numerator open parentheses x plus 2 close parentheses open parentheses x plus 1 close parentheses open parentheses x minus 1 close parentheses open parentheses x minus 2 close parentheses over denominator 4 end fraction
l subscript 3 open parentheses x close parentheses equals fraction numerator open parentheses x space minus space x subscript 0 close parentheses. open parentheses x space minus space x subscript 1 close parentheses. space open parentheses x space minus space x subscript 2 close parentheses. open parentheses x space minus space x subscript 4 close parentheses over denominator open parentheses x subscript 3 minus x subscript 0 close parentheses. space open parentheses x subscript 3 minus x subscript 1 close parentheses. open parentheses x subscript 3 minus x subscript 2 close parentheses. open parentheses x subscript 3 minus x subscript 4 close parentheses end fraction space equals space fraction numerator open parentheses x plus 2 close parentheses open parentheses x plus 1 close parentheses open parentheses x close parentheses open parentheses x minus 2 close parentheses over denominator negative 6 end fraction
l subscript 4 open parentheses x close parentheses equals fraction numerator open parentheses x space minus space x subscript 0 close parentheses. open parentheses x space minus space x subscript 1 close parentheses. space open parentheses x space minus space x subscript 2 close parentheses. open parentheses x space minus space x subscript 3 close parentheses over denominator open parentheses x subscript 4 minus x subscript 0 close parentheses. space open parentheses x subscript 4 minus x subscript 1 close parentheses. open parentheses x subscript 4 minus x subscript 2 close parentheses. open parentheses x subscript 4 minus x subscript 3 close parentheses end fraction space equals space fraction numerator open parentheses x plus 2 close parentheses open parentheses x plus 1 close parentheses open parentheses x close parentheses open parentheses x minus 1 close parentheses over denominator 24 end fraction

p(x) = sum from i space equals space 0 to n of space l subscript i open parentheses x close parentheses f open parentheses x subscript i close parentheses= space 2 space fraction numerator open parentheses x space plus 1 close parentheses open parentheses x close parentheses open parentheses x minus 1 close parentheses open parentheses x minus 2 close parentheses over denominator 24 end fraction plus 14 space fraction numerator negative open parentheses x space plus 2 close parentheses open parentheses x close parentheses open parentheses x minus 1 close parentheses open parentheses x space minus 2 close parentheses over denominator 6 end fraction plus space 4 fraction numerator open parentheses x plus 2 close parentheses open parentheses x plus 1 close parentheses open parentheses x minus 1 close parentheses open parentheses x minus 2 close parentheses over denominator 4 end fraction plus space
2 fraction numerator open parentheses x plus 2 close parentheses open parentheses x plus 1 close parentheses open parentheses x close parentheses open parentheses x minus 2 close parentheses over denominator negative 6 end fraction plus space 2 fraction numerator open parentheses x plus 2 close parentheses open parentheses x plus 1 close parentheses open parentheses x close parentheses open parentheses x minus 1 close parentheses over denominator 24 end fraction=

= space fraction numerator open parentheses x space plus 1 close parentheses open parentheses x close parentheses open parentheses x minus 1 close parentheses open parentheses x minus 2 close parentheses over denominator 12 end fraction plus 7 fraction numerator negative open parentheses x space plus 2 close parentheses open parentheses x close parentheses open parentheses x minus 1 close parentheses open parentheses x space minus 2 close parentheses over denominator 3 end fraction plus space open parentheses x plus 2 close parentheses open parentheses x plus 1 close parentheses open parentheses x minus 1 close parentheses open parentheses x minus 2 close parentheses plus space
minus fraction numerator open parentheses x plus 2 close parentheses open parentheses x plus 1 close parentheses open parentheses x close parentheses open parentheses x minus 2 close parentheses over denominator 3 end fraction plus space fraction numerator open parentheses x plus 2 close parentheses open parentheses x plus 1 close parentheses open parentheses x close parentheses open parentheses x minus 1 close parentheses over denominator 12 end fraction

= 

21. (Continuation) Find a polynomial that takes the values shown in the preceding problem and has at x = 3 the value 10. Hint: Add a suitable polynomial to the p(x) of the previous problem.

Answer:

q(x) = p(x) + c(x+2)(x+1)(x)(x-1)(x-2)

q(3) = p(3) + 120c

We have p(3) = -38 and q(3) = 10

Then q(3) = p(3) + 120c

10 = -38 + 120c then c = 2/5

Now: q(x) = p(x) + c(x+2)(x+1)(x)(x-1)(x-2)

q open parentheses x close parentheses equals space 4 space minus space 8 x space plus space fraction numerator 11 x squared over denominator 2 end fraction plus space 2 x cubed minus fraction numerator 3 x to the power of 4 over denominator 2 end fraction plus 2 over 5 open parentheses x plus 2 close parentheses open parentheses x plus 1 close parentheses x open parentheses x minus 1 close parentheses open parentheses x minus 2 close parentheses
H e n c e comma space q open parentheses x close parentheses space equals space 4 space minus 32 over 5 x plus 11 over 2 x squared minus 3 over 2 x to the power of 4 plus 2 over 5 x to the power of 5



23. Form a divided-difference table for the following and explain what happened



Answer:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| x | f[ ] | f[ , ] | f[ , , ] | f[ , , , ] |
| 1 | 3 | f[1,2]=(5-3)/(2-1)=2 | f[1,2,3] = (0-2)/(3-1) = -1 | f[1,2,3,1] = (1+1)/(1-1) = undefined |
| 2 | 5 | f[2,3] = (5-5)/(3-2) = 0 | f[2,3,1]= (-1-0)(1-2) = 1 |  |
| 3 | 5 | f[3,1] = (7-5)/(1-3) = -1 |  |  |
| 1 | 7 |  |  |  |

We cannot division by zero since the nodes are not unique. Interpolating polynomials are functions, but the give data set is not from a function, but a relation

34. Write the Lagrange form (1) of the interpolating polynomial of degree at most 2 that interpolates f (x) at x0, x1, and x2, where x0 < x1 < x2.

**Answer:**

p(x) = sum from i space equals space 0 to n of space l subscript i open parentheses x close parentheses f open parentheses x subscript i close parentheses

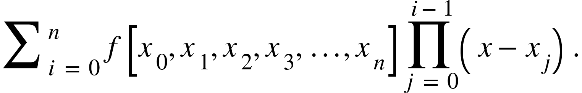
li(x)= stack product fraction numerator x space minus space x subscript j over denominator x subscript i space minus space x subscript j end fraction with j not equal to i space comma j space equals space 0 below and n on top

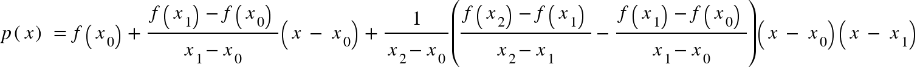

l subscript 0 open parentheses x close parentheses equals fraction numerator open parentheses x space minus space x subscript 1 close parentheses. open parentheses x space minus space x subscript 2 close parentheses over denominator open parentheses x subscript 0 minus x subscript 1 close parentheses. space open parentheses x subscript 0 minus x subscript 2 close parentheses end fraction space
l subscript 1 open parentheses x close parentheses equals fraction numerator open parentheses x space minus space x subscript 0 close parentheses. open parentheses x space minus space x subscript 2 close parentheses over denominator open parentheses x subscript 1 minus x subscript 0 close parentheses. space open parentheses x subscript 1 minus x subscript 2 close parentheses end fraction space
l subscript 2 open parentheses x close parentheses equals fraction numerator open parentheses x space minus space x subscript 0 close parentheses. open parentheses x space minus space x subscript 1 close parentheses over denominator open parentheses x subscript 2 minus x subscript 0 close parentheses. space open parentheses x subscript 2 minus x subscript 1 close parentheses end fraction space


p open parentheses x close parentheses equals space f open parentheses x subscript 0 close parentheses l subscript 0 open parentheses x close parentheses plus f open parentheses x subscript 1 close parentheses l subscript 1 open parentheses x close parentheses plus f open parentheses x subscript 2 close parentheses l subscript 2 open parentheses x close parentheses
p open parentheses x close parentheses equals f open parentheses x subscript 0 close parentheses fraction numerator open parentheses x space minus space x subscript 1 close parentheses. open parentheses x space minus space x subscript 2 close parentheses over denominator open parentheses x subscript 0 minus x subscript 1 close parentheses. space open parentheses x subscript 0 minus x subscript 2 close parentheses end fraction space plus f open parentheses x subscript 1 close parentheses fraction numerator open parentheses x space minus space x subscript 0 close parentheses. open parentheses x space minus space x subscript 2 close parentheses over denominator open parentheses x subscript 1 minus x subscript 0 close parentheses. space open parentheses x subscript 1 minus x subscript 2 close parentheses end fraction space plus f open parentheses x subscript 2 close parentheses fraction numerator open parentheses x space minus space x subscript 0 close parentheses. open parentheses x space minus space x subscript 1 close parentheses over denominator open parentheses x subscript 2 minus x subscript 0 close parentheses. space open parentheses x subscript 2 minus x subscript 1 close parentheses end fraction space


35. (Continuation) Write the Newton form of the interpolating polynomial p2(x), and show that it is equivalent to the Lagrange form.

Answer:

p(x) = 



p open parentheses x close parentheses space equals space f open parentheses x subscript 0 close parentheses plus fraction numerator f open parentheses x subscript 1 close parentheses minus f open parentheses x subscript 0 close parentheses over denominator x subscript 1 minus x subscript 0 end fraction open parentheses x space minus space x subscript 0 close parentheses plus fraction numerator 1 over denominator x subscript 2 minus x subscript 0 end fraction open parentheses fraction numerator f open parentheses x subscript 2 close parentheses minus f open parentheses x subscript 1 close parentheses over denominator x subscript 2 minus x subscript 1 end fraction minus fraction numerator f open parentheses x subscript 1 close parentheses minus f open parentheses x subscript 0 close parentheses over denominator x subscript 1 minus x subscript 0 end fraction close parentheses open parentheses x space minus space x subscript 0 close parentheses open parentheses x space minus space x subscript 1 close parentheses
p open parentheses x close parentheses space equals space space f open parentheses x subscript 0 close parentheses open parentheses 1 space minus fraction numerator open parentheses x space minus space x subscript 0 close parentheses over denominator x subscript 1 minus x subscript 0 end fraction plus fraction numerator open parentheses x space minus space x subscript 0 close parentheses open parentheses x space minus space x subscript 1 close parentheses over denominator x subscript 1 minus x subscript 0 end fraction space close parentheses space plus f open parentheses x subscript 1 close parentheses open parentheses fraction numerator open parentheses x space minus space x subscript 0 close parentheses over denominator x subscript 1 minus x subscript 0 end fraction minus fraction numerator open parentheses x space minus space x subscript 0 close parentheses open parentheses x space minus space x subscript 1 close parentheses over denominator open parentheses x subscript 2 minus x subscript 1 close parentheses open parentheses x subscript 2 minus x subscript 0 close parentheses end fraction minus fraction numerator open parentheses x space minus space x subscript 0 close parentheses open parentheses x space minus space x subscript 1 close parentheses over denominator open parentheses x subscript 1 minus x subscript 0 close parentheses open parentheses x subscript 2 minus x subscript 0 close parentheses end fraction close parentheses plus space f open parentheses x subscript 2 close parentheses open parentheses fraction numerator open parentheses x space minus space x subscript 0 close parentheses open parentheses x space minus space x subscript 1 close parentheses over denominator open parentheses x subscript 2 minus x subscript 1 close parentheses open parentheses x subscript 2 minus x subscript 0 close parentheses end fraction close parentheses

p open parentheses x close parentheses equals f open parentheses x subscript 0 close parentheses fraction numerator open parentheses x space minus space x subscript 1 close parentheses. open parentheses x space minus space x subscript 2 close parentheses over denominator open parentheses x subscript 0 minus x subscript 1 close parentheses. space open parentheses x subscript 0 minus x subscript 2 close parentheses end fraction space plus f open parentheses x subscript 1 close parentheses fraction numerator open parentheses x space minus space x subscript 0 close parentheses. open parentheses x space minus space x subscript 2 close parentheses over denominator open parentheses x subscript 1 minus x subscript 0 close parentheses. space open parentheses x subscript 1 minus x subscript 2 close parentheses end fraction space plus f open parentheses x subscript 2 close parentheses fraction numerator open parentheses x space minus space x subscript 0 close parentheses. open parentheses x space minus space x subscript 1 close parentheses over denominator open parentheses x subscript 2 minus x subscript 0 close parentheses. space open parentheses x subscript 2 minus x subscript 1 close parentheses end fraction space


From Lagrange:

q open parentheses x close parentheses equals space f open parentheses x subscript 0 close parentheses l subscript 0 open parentheses x close parentheses plus f open parentheses x subscript 1 close parentheses l subscript 1 open parentheses x close parentheses plus f open parentheses x subscript 2 close parentheses l subscript 2 open parentheses x close parentheses
q open parentheses x close parentheses equals f open parentheses x subscript 0 close parentheses fraction numerator open parentheses x space minus space x subscript 1 close parentheses. open parentheses x space minus space x subscript 2 close parentheses over denominator open parentheses x subscript 0 minus x subscript 1 close parentheses. space open parentheses x subscript 0 minus x subscript 2 close parentheses end fraction space plus f open parentheses x subscript 1 close parentheses fraction numerator open parentheses x space minus space x subscript 0 close parentheses. open parentheses x space minus space x subscript 2 close parentheses over denominator open parentheses x subscript 1 minus x subscript 0 close parentheses. space open parentheses x subscript 1 minus x subscript 2 close parentheses end fraction space plus f open parentheses x subscript 2 close parentheses fraction numerator open parentheses x space minus space x subscript 0 close parentheses. open parentheses x space minus space x subscript 1 close parentheses over denominator open parentheses x subscript 2 minus x subscript 0 close parentheses. space open parentheses x subscript 2 minus x subscript 1 close parentheses end fraction space


Now we can consider p(x) = q(x).

Hence, Newton form of the interpolating polynomial p2(x), and it is equivalent to the Lagrange form.

**Section 4.2: 6, 7, 10, 11**

6. How accurately can we determine sin x by linear interpolation, given a table of sin x to ten decimal places, for x in [0, 2] with h = 0.01?

Answer:

Text

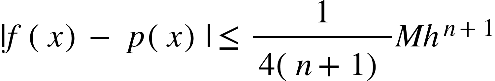
Description automatically generated

f(x) = sin x

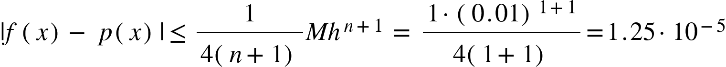
f’(x) = cos x

f’’(x) = -sin x

|f’’(x)| ≤1 = M



h = (b – a)/n is space between nodes. Since we want to approximate f(x) = sin x linear interpolation. We have n =1

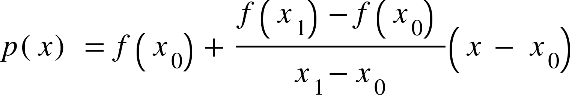


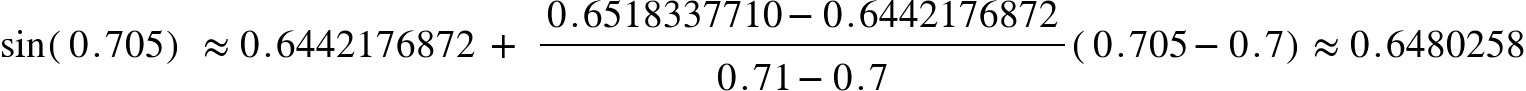
7. (Continuation) Given the data

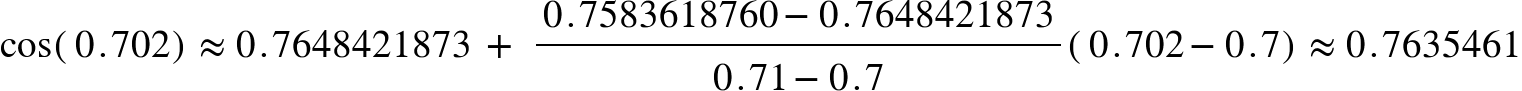
Table

Description automatically generated

sin open parentheses 0.705 close parentheses space almost equal to 0.6442176872 space plus space fraction numerator 0.6518337710 minus 0.6442176872 over denominator 0.71 minus 0.7 end fraction open parentheses 0.705 minus 0.7 close parentheses almost equal to 0.6480258**:**







Using direct trigonometry computation:

sin(0.705) = 0.6480338

cos(0.702) = 0.7635522

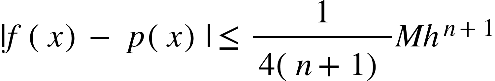
Error on sin = |0.6480338 – 0.6480258| = 8.1\*10-6

Error on cos = |0.7635461-0.7635522| = 6.123\*10-6.

10. Let the function f (x) = ln x be approximated by an interpolation polynomial of degree 9 with ten nodes uniformly distributed in the interval [1, 2]. What bound can be placed on the error?

**Answer:**

n = 9



Since f(x) = lnx and x [1.2]

|f’(x)| = |x-1|

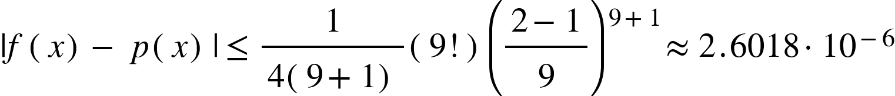
|f’’(x)| = |-x-2|

|f’’’(x)| = |2x-3|

….

|f(10)(x)|= |9!x-10|

With x-10 ≤1 then |f(10)x| ≤ 9! = M



11. In the first theorem on interpolation errors, show that if x0 < x1 < ··· < xn and x0 < x < xn, then x0 <ξ< xn.

**Answer:**

Define x0 < x < xn,

Text

Description automatically generated

Text

Description automatically generated with low confidence

Where p(t) is the polynomial degree at most n that interpolates n+ 1 points x0throughxn evaluated with f(x).

Note also that ϕ takes the value 0 at the n + 2 points x0, x1,..., xn, and x. Now invoke Rolle’s Theorem, ∗ which states that between any two roots of ϕ, there must occur a root of ϕ . Thus, ϕ has at least n + 1 roots. By similar reasoning, ϕ has at least n roots, ϕ has at least n − 1 roots, and so on. Finally, it can be inferred that ϕ(n+1) must have at least one root. Let ξ be a root of ϕ(n+1) . All the roots being counted in this argument are in (a, b). Thus, ξ between x0 and xn. Thus x0 < ξ < xn

**Computing Exercises**

Section 4.2: 1, 2

1. Using 21 equally spaced nodes on the interval [−5, 5], find the interpolating polynomial p of degree 20 for the function f (x) = (x 2 +1)−1. Print the values of f (x) and p(x) at 41 equally spaced points, including the nodes. Observe the large discrepancy between f (x) and p(x).

Answer:

Mathlab code:

close all

clc

x = linspace(-5,5,21);

fx = 1./(x.^2+1);

n = 20;

p = polyfit(x,fx,n); % interpolating polynomial p of degree 20 for the function f (x)

x = linspace(-5,5,41);

fx = 1./(x.^2+1);

px = polyval(p,x); % Count p(x)

fprintf("\nDisplay lists fx || px\n")

[fx' px']

figure;plot(x,fx)

hold on

plot(x,px)

legend('f(x)','p(x)')

ylim([-1 4.5])

Display:

Display lists fx || px

ans =

0.0385 0.0385

0.0424 -39.9524

0.0471 0.0471

0.0525 3.4550

0.0588 0.0588

0.0664 -0.4471

0.0755 0.0755

0.0865 0.2024

0.1000 0.1000

0.1168 0.0807

0.1379 0.1379

0.1649 0.1798

0.2000 0.2000

0.2462 0.2384

0.3077 0.3077

0.3902 0.3951

0.5000 0.5000

0.6400 0.6368

0.8000 0.8000

0.9412 0.9425

1.0000 1.0000

0.9412 0.9425

0.8000 0.8000

0.6400 0.6368

0.5000 0.5000

0.3902 0.3951

0.3077 0.3077

0.2462 0.2384

0.2000 0.2000

0.1649 0.1798

0.1379 0.1379

0.1168 0.0807

0.1000 0.1000

0.0865 0.2024

0.0755 0.0755

0.0664 -0.4471

0.0588 0.0588

0.0525 3.4550

0.0471 0.0471

0.0424 -39.9524

0.0385 0.0385

Chart, line chart, histogram

Description automatically generated

**Another way to code:**

clc

f=@(x) (x.^2+1).^(-1);

%create 21 point

x=linspace(-5,5,21);

y=fun(x);

%function to interpolate values

array=coef(x,y);

%create 41 point

x\_p=linspace(-5,5,41);

fprintf(' Index |x\_values |Actual\_values |interpolated values |abs difference\n');

for i=1:1:length(x\_p)

x\_p\_Index=x\_p(i);

%interpolate values

inter\_value=Evaluation(x,array,x\_p\_Index);

Actual\_value=fun(x\_p\_Index);

% Display

fprintf('%5.0f %10.5f %15.5f %15.5f %15.5f\n',i,x\_p\_Index,Actual\_value,inter\_value,abs(Actual\_value-inter\_value));

end

% Get funtion coef mean a given function to interpolate values

function[array]=coef(x,y)

n=length(x);

m=length(y);

if n~=m,error('same length vector applicable');end

F=zeros(n,n);

F(:,1)=y';

for j=2:n

for i=1:(n-j+1)

F(i,j)=(F(i+1,j-1)-F(i,j-1))/(x(i+j-1)-x(i));

end

end

array=F(1,:);

end

function inter\_value=Evaluation(x,array,x\_p\_Index)

%Performs approximation of the values

z=length(array);

sum=0;

for i=1:z

value\_prodx=1;

for j=1:i-1

value\_prodx=value\_prodx\*(x\_p\_Index-x(j));

end

sum=sum+array(i)\*value\_prodx;

end

inter\_value=sum;

end

**Display**

Index |x\_values |Actual\_values |interpolated values |abs difference  
 1 -5.00000 0.03846 0.03846 0.00000  
 2 -4.75000 0.04244 -39.95245 39.99489  
 3 -4.50000 0.04706 0.04706 0.00000  
 4 -4.25000 0.05246 3.45496 3.40250  
 5 -4.00000 0.05882 0.05882 0.00000  
 6 -3.75000 0.06639 -0.44705 0.51344  
 7 -3.50000 0.07547 0.07547 0.00000  
 8 -3.25000 0.08649 0.20242 0.11594  
 9 -3.00000 0.10000 0.10000 0.00000  
 10 -2.75000 0.11679 0.08066 0.03613  
 11 -2.50000 0.13793 0.13793 0.00000  
 12 -2.25000 0.16495 0.17976 0.01481  
 13 -2.00000 0.20000 0.20000 0.00000  
 14 -1.75000 0.24615 0.23845 0.00771  
 15 -1.50000 0.30769 0.30769 0.00000  
 16 -1.25000 0.39024 0.39509 0.00485  
 17 -1.00000 0.50000 0.50000 0.00000  
 18 -0.75000 0.64000 0.63676 0.00324  
 19 -0.50000 0.80000 0.80000 0.00000  
 20 -0.25000 0.94118 0.94249 0.00131  
 21 0.00000 1.00000 1.00000 0.00000  
 22 0.25000 0.94118 0.94249 0.00131  
 23 0.50000 0.80000 0.80000 0.00000  
 24 0.75000 0.64000 0.63676 0.00324  
 25 1.00000 0.50000 0.50000 0.00000  
 26 1.25000 0.39024 0.39509 0.00485  
 27 1.50000 0.30769 0.30769 0.00000  
 28 1.75000 0.24615 0.23845 0.00771  
 29 2.00000 0.20000 0.20000 0.00000  
 30 2.25000 0.16495 0.17976 0.01481  
 31 2.50000 0.13793 0.13793 0.00000  
 32 2.75000 0.11679 0.08066 0.03613  
 33 3.00000 0.10000 0.10000 0.00000  
 34 3.25000 0.08649 0.20242 0.11594  
 35 3.50000 0.07547 0.07547 0.00000  
 36 3.75000 0.06639 -0.44705 0.51344  
 37 4.00000 0.05882 0.05882 0.00000  
 38 4.25000 0.05246 3.45496 3.40250  
 39 4.50000 0.04706 0.04706 0.00000  
 40 4.75000 0.04244 -39.95245 39.99489  
 41 5.00000 0.03846 0.03846 0.00000

2. (Continuation) Perform the experiment in the preceding computer problem, using Chebyshev nodes xi = 5 cos(iπ/20), where 0 ≤i ≤20, and nodes xi = 5 cos[(2i + 1)π/42], where 0 ≤i ≤20. Record your conclusions.

**Answer:**

Mathlab code:

clc

f=@(x) (x.^2+1).^(-1);

i = 0:1:20;

c\_p = 5\*cos(i\*pi/20);

x\_p = linspace(-5,5,20);

ChebyshevInterpolate(f,c\_p,x\_p);

function ChebyshevInterpolate(fun,c\_p,x\_p)

n =length(c\_p);

T = zeros(n,n);

F = zeros(n,1);

for i = 1:1:n

c\_p\_index = c\_p(i);

for j = 1:1:n

if j ==1

T(i,j)=1;

else

T(i,j) = chebyshevT(j-1,c\_p\_index);

end

end

F(i) = fun(c\_p\_index);

end

coeff = T\F;

fprintf(' Index Exact Interpolated Difference(Error)\n');

for i = 1:1:length(x\_p)

sum = coeff(1);

x\_index = x\_p(i);

for j = 2:1:n

Tj = chebyshevT(j-1,x\_index);

sum = sum + coeff(j)\*Tj;

end

y\_p\_c = sum;

y\_p\_e = fun(x\_index);

fprintf('%5.0f %15.5f %15.5f %15.5f\n',i,y\_p\_e,y\_p\_c,abs(y\_p\_e - y\_p\_c));

end

end

Display

Index Exact Interpolated Difference(Error)

1 0.03846 0.03846 0.00000

2 0.04759 0.04701 0.00058  
 3 0.06031 0.05704 0.00326

4 0.07872 0.08303 0.00431  
 5 0.10662 0.10461 0.00200

6 0.15130 0.14629 0.00501  
 7 0.22762 0.24132 0.01370

8 0.36613 0.35211 0.01402  
 9 0.61604 0.61552 0.00052

10 0.93523 0.94311 0.00788  
 11 0.93523 0.94311 0.00788

12 0.61604 0.61552 0.00052  
 13 0.36613 0.35211 0.01402

14 0.22762 0.24132 0.01370  
 15 0.15130 0.14629 0.00501

16 0.10662 0.10461 0.00200  
 17 0.07872 0.08303 0.00431

18 0.06031 0.05704 0.00326  
 19 0.04759 0.04701 0.00058

20 0.03846 0.03846 0.00000